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ELASTIC EQUILIBRIUM OF AN ANISOTROPIC CYLINDER WITH LONGITUDINAL CAVITIES SUBJECTED TO AXIAL LOADS

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The stresses and displacements in a long anisotropic cylinder with longitudinal cavities are determined. The problem reduces to seeking an analytic function of a complex variable which is determined in a domain obtained by an affine transformation from the domain of the cylinder cross section. Boundary conditions and a general representation are obtained for the function mentioned.

A long cylinder attenuated by longitudinal cavities, fabricated from a homogeneous linearly-elastic material having a plane of elastic symmetry perpendicular to the cylinder axis at each point is considered. The cylinder is clamped to a rigid mass without tension along the outer surface. Axial tangential forces which do not vary along the cylinder axis are applied to the surfaces of the cavities. Moreover, axial gravitational forces act on the cylinder.

We introduce a rectangular xyz-coordinate system with the z-axis directed downward along the cylinder axis. Let S be a domain occupied by a cross section in the xy-plane, L_0 and L_k are its outer and inner contours ($k = 1, 2, \ldots, N$), γ is the specific gravity of the material, and $\tau_k(s)$ is the intensity of the external forces applied to the k-th cavity.

Following Moskvitin [1] and using the Hooke's law equation in the form written in [2], we find an equation for the axial displacement function

$$A_{44}Dw = A_{55}\frac{\partial^2 w}{\partial x^2} + 2A_{45}\frac{\partial^2 w}{\partial x\,\partial y} + A_{44}\frac{\partial^2 w}{\partial y^2} = -\gamma \tag{1}$$

Here A_{44}, A_{45}, A_{55} are elastic constants of the material. We represent the general solution of (1) as in [2]

$$A_{44}w = w_1 + 2\operatorname{Re}\Phi(z_1)$$

Here w_1 is some particular solution of the inhomogeneous equation $Dw_1 = -\gamma$, and $\Phi(z_1)$ is an analytic function of the auxiliary complex variable $z_1 = x_1 + iy_1$, where

$$x_1 = x + \alpha y, \quad y_1 = \beta y \tag{2}$$

$$\alpha = -A_{45} / A_{44}, \quad \beta = \sqrt{\beta_1 - \alpha^2}, \quad \beta_1 = A_{55} / A_{44}$$

If the function $\Phi(z_1)$ has been found, the stresses can be evaluated by the formulas

$$\tau_{xz} = \tau_{xz}{}^1 + 2\beta \operatorname{Im} [(\alpha + i\beta)\Phi'(z_1)]$$

$$\tau_{yz} = \tau_{yz}{}^1 - 2\beta \operatorname{Im}\Phi'(z_1)$$

$$\left(\tau_{xz}^1 = \beta_1 \frac{\partial w_1}{\partial x} - \alpha \frac{\partial w_1}{\partial y}, \ \tau_{yz}^1 = \frac{\partial w_1}{\partial y} - \alpha \frac{\partial w_1}{\partial x}\right)$$

Here $\tau_{xz}{}^1$, $\tau_{yz}{}^1$ are stress components corresponding to the particular solution w_1. They are defined by the equalities in the parentheses. The remaining stress components as well as the transverse displacements u, v are zero.

The axial displacements and stresses must be single-valued functions in the domain S. Any coaxial cylinder, imagined isolated in the cylinder under consideration, must be in equilibrium. By satisfying these requirements, we obtain the following general expression for the function introduced:

$$\Phi(z_1) = \sum_{k=1}^{N} \frac{T_k - \gamma\Omega_k}{4\pi\beta} \ln(z_1 - z_{1k}) + \Phi_0(z_1)$$

Here $\Phi_0(z_1)$ is an arbitrary function, homomorphic in the domain S_1 and obtained by the affine transformations (2) from the domain S . The constant T_k is the magnitude of the principal vector of the external forces $\tau_k(s)$ applied to the k-th cavity, and Ω_k is the area of the domain S_k bounded by the contour L_k. Here z_{1k} denotes the affix of points lying within the boundary contour L_{1k} of the auxiliary domain S^1.

From the boundary conditions

$$w = 0 \quad \text{on } L_0$$

$$\tau_{nz} = \tau_k(s) \qquad \text{on } L_k$$

we obtain the following boundary conditions for the required function $\Phi(z_1)$

$$\Phi(t_{10}) + \overline{\Phi(t_{10})} = -w_1 \tag{3}$$

$$\Phi(t_{1k}) - \overline{\Phi(t_{1k})} = \frac{i}{\beta} \int_0^s (\tau_{nz}^1 - \tau_k)\, ds + iC_k$$

The quantities t_{10} and t_{1k} in these equalities are affixes of points of the external and internal contours L_{10} and L_{1k} of the domain S^1. Values of the constants of integration C_k do not influence determination of the displacements and stresses, hence they need not be determined. For the same reason, the pure imaginary constant component in the representation of the function $\Phi(z_1)$ can be discarded.

Therefore, the problem posed reduces to seeking an analytic function of a complex variable which is defined in the auxiliary domain S^1 and satisfies the conditions (3) on its boundaries.

It is expedient to determine the required function by approximate methods for specific kinds of domains of the cylinder cross section.

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